

5-3 Monomial Factors of Polynomials

Objective: To divide polynomials by monomials and to find monomial factors of polynomials.

Vocabulary

Divisible One polynomial is (evenly) divisible by another polynomial if the quotient is also a polynomial. Example 1b shows that $27uv - 36v$ is divisible by $9v$.

Greatest monomial factor of a polynomial The GCF of the terms of the polynomial. In Example 3, the GCF of $3x^2 + 12x$ is $3x$.

Example 1 Divide: a. $\frac{6m + 36}{6}$ b. $\frac{27uv - 36v}{9v}$

Solution Divide each term of the polynomial by the monomial. Then add the results.

$$\begin{aligned} \text{a. } \frac{6m + 36}{6} &= \frac{6m}{6} + \frac{36}{6} & \text{b. } \frac{27uv - 36v}{9v} &= \frac{27uv}{9v} - \frac{36v}{9v} \\ &= m + 6 & &= 3u - 4 \end{aligned}$$

Example 2 Divide: $\frac{2x^4 - 8x^3y + 4x^2y^2}{-2x^2}$

Solution
$$\frac{2x^4 - 8x^3y + 4x^2y^2}{-2x^2} = \frac{2x^4}{-2x^2} - \frac{8x^3y}{-2x^2} + \frac{4x^2y^2}{-2x^2} = -x^2 + 4xy - 2y^2$$

Divide. Assume that no denominator equals zero.

- $\frac{4a + 12}{4} a + 3$
- $\frac{10a - 15}{5} 2a - 3$
- $\frac{20n - 16}{4} 5n - 4$
- $\frac{6x + 9y + 12}{3} 2x + 3y + 4$
- $\frac{2m - 4n + 6}{2} m - 2n + 3$
- $\frac{x^3 - 4x^2 + 6x}{x} x^2 - 4x + 6$
- $\frac{8xy - 12x^2}{4x} 2y - 3x$
- $\frac{5a - 10a^2 - 15a^3}{5a} 1 - 2a - 3a^2$
- $\frac{12y - 18y^2 - 6y^3}{6y} 2 - 3y - y^2$
- $\frac{4x^2 - 12x - 8}{4} x^2 - 3x - 2$
- $\frac{27y^4 + 18y^3 - 36y^2}{9y^2} 3y^2 + 2y - 4$
- $\frac{6u^3 + 4u^2 - 2u}{2u} 3u^2 + 2u - 1$
- $\frac{12r^4 - 9r^3 - 6r^2}{-3r^2} -4r^2 + 3r + 2$
- $\frac{5m^3 + 8m^4 - 3m^5}{-m^3} -5 - 8m + 3m^2$
- $\frac{xy^3 + x^3y}{xy} y^2 + x^2$
- $\frac{8ab^2 - 12a^2b}{4ab} 2b - 3a$

5-3 Monomial Factors of Polynomials (continued)

Example 3 Factor $3x^2 + 12x$

- Solution**
- The greatest monomial factor of $3x^2 + 12x$ is $3x$.
 - Divide to find the other factor:
$$\frac{3x^2 + 12x}{3x} = \frac{3x^2}{3x} + \frac{12x}{3x} = x + 4$$
 - $3x^2 + 12x = 3x(x + 4)$

Example 4 Factor $6x^5 - 4x^3 + 8x$

- Solution**
- The greatest monomial factor of $6x^5 - 4x^3 + 8x$ is $2x$.
 - Divide to find the other factor:
$$\frac{6x^5 - 4x^3 + 8x}{2x} = \frac{6x^5}{2x} - \frac{4x^3}{2x} + \frac{8x}{2x} = 3x^4 - 2x^2 + 4$$
 - $6x^5 - 4x^3 + 8x = 2x(3x^4 - 2x^2 + 4)$

Factor.

- $21a^3 - 14a^2$ **$7a^2(3a - 2)$**
- $4x^3 + 32x$ **$4x(x^2 + 8)$**
- $9x^2 + 36x$ **$9x(x + 4)$**
- $21c^3 - 14c$ **$7c(3c^2 - 2)$**
- $10a - 35b + 15$ **$5(2a - 7b + 3)$**
- $16x - 12y + 24$ **$4(4x - 3y + 6)$**
- $8p - 4q + 12$ **$4(2p - q + 3)$**
- $3x - 6y + 12$ **$3(x - 2y + 4)$**
- $9x - 6y + 36$ **$3(3x - 2y + 12)$**
- $15a - 20b + 10$ **$5(3a - 4b + 2)$**
- $3a^3 + 6a^2 - 12a$ **$3a(a^2 + 2a - 4)$**
- $10x^3 - 5x^2 + 20x$ **$5x(2x^2 - x + 4)$**
- $5y^3 - 10y^2 + 15y$ **$5y(y^2 - 2y + 3)$**
- $18x^3 - 6x^2 + 24x$ **$6x(3x^2 - x + 4)$**
- $8ab^2 - 12a^2b$ **$4ab(2b - 3a)$**
- $3a^2b^2 + 18ab$ **$3ab(ab + 6)$**
- $6y^3 - 24y^2 - 12y$ **$6y(y^2 - 4y - 2)$**
- $20y^4 + 35y^3 + 15y^2$ **$5y^2(4y^2 + 7y + 3)$**

Mixed Review Exercises

Simplify.

- $6n^2\left(\frac{1}{6}n^2\right) n^4$
- $8x^2\left(\frac{3}{4}x^3\right) 6x^5$
- $3a^2 - 6ac^2 + 4a^2 - 5ac^2$
- $\frac{5x^3y}{10x^2y^2} \frac{x}{2y}$
- $24 \div \left(-\frac{1}{3}\right) - 72$
- $\frac{(3a^2)^3}{a^4} 27a^2$
- $(3a)^4 81a^4$
- $6(3^2 - 1) + 2^3 56$
- $(x - 1)(x^2 + 2x + 3)$
- $(m - 3)(m + 4)$
- $(3a + 2)(5a - 3)$
- $7a^2 - 11ac^2$
- $x^3 + x^2 + x - 3$
- $m^2 + m - 12$
- $15a^2 + a - 6$
- $12p^2 - 20pq + 3q^2$